Mathematics for Supply Chain

Msc Supply Chain & Purchasing

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A copy of your work is due on Monday, November 28, *i.e.*, for the last class.
The exercises are all independent and are designed to apply the concepts learned in class. You are free to use excel to perform your calculations. If so, please specify on your copy the excel formula used to obtain the result.

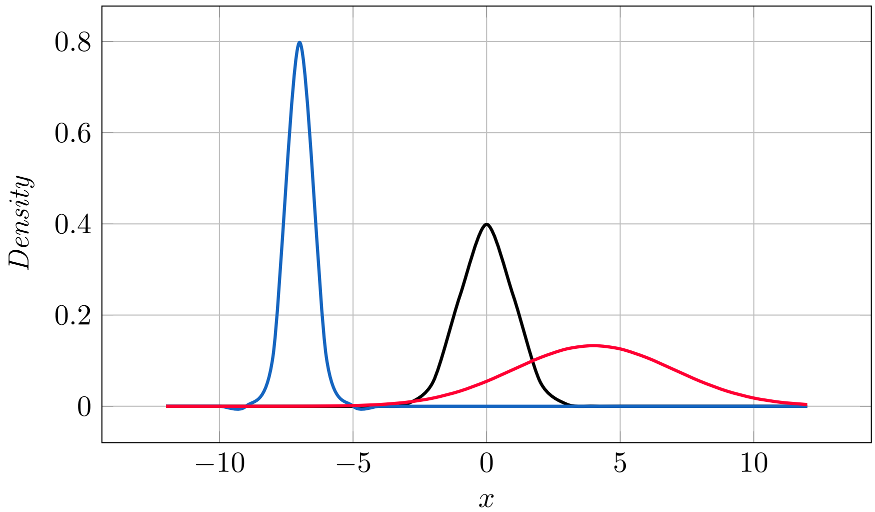
1 Generalities on the Normal Distribution

Exercice 1.1 (Generalities on Continuous Random Variables)**.**

We are working on the properties of the normal distribution.

1. *A continuous random variable* X *has a normal distribution with mean* 73*. The probability that* X *takes a value greater than* 80 *is* 0.212*. Find the probability that* X *takes a value less than* 66 *. Sketch the density curve with relevant regions shaded to illustrate the computation.*

2. *The figure below represent* 3 *different normal distributions with mean values equal to* $0, -7$ *and* 3 *with standard deviations equal to* $1, 3$ *and* 0.5*.*



Associate each curve to its couple of parameters (μ, σ)*.*

3. *Financial market prices are subject to wide variations over time, and it is assumed that the values taken by the latter over time are distributed according to a centered normal distribution whose mean is equal to* 6800 *points. We know that* 22% *of the values taken by the CAC 40 price are below* 6500 *points and that* 10% *of the values taken are between* 7000 *and* 7100*. In what proportion does the share price take values between* 6500 *and* 7000*? You can use a graph to solve this question.*

Exercice 1.2 (Applications)**.** *We are now working on several applications with the normal distribution.*

1. *The systolic blood pressure* X *of adults in a region is normally distributed with mean equal* 112 *and standard deviation* 15*. A person is considered "prehypertensive" if his*

systolic blood pressure is between 120 *and* 130. *Find the probability that the blood pressure of a randomly selected person is prehypertensive.*

2. *The length of time that the battery in Mathew cell phone will hold enough charge to operate acceptably is normally distributed with mean* 25.6 *hours and standard deviation* 0.32 *hour. Mathew forgot to charge his phone yesterday, so that at the moment she first wishes to use it today it has been* 26 *hours* 18 *minutes since the phone was last fully charged. Find the probability that the phone will operate properly.*

Exercice 1.3 (Find the parameters of a Normal Distribution)**.**

A company carried out an analysis of the various costs involved in developing new marketing strategies. It estimated that 10% *of development costs were in excess of $* 75,000 *and that* 25% *were under $* 50,000. *Assuming that X is a random variable following a normal distribution, determine the parameters of this distribution.*

2 Sampling estimation and Confidence Interval

Exercice 2.1 (Basics on Sampling Estimation)**.**

Answer to the basic following questions:

1. *Random samples of size* 225 *are drawn from a population with mean 100 and standard deviation* 20. *Find the mean and standard deviation of the sample mean.*

2. *Random samples of size* 64 *are drawn from a population with mean* 32 *and standard deviation* 5. *Find the mean and standard deviation of the sample mean.*

3. *A population has mean* 75 *and standard deviation* 12.

   (a) *Random samples of size* 121 *are taken. Find the mean and standard deviation of the sample mean.*

   (b) *How would the answers to part (a) change if the size of the samples were* 400 *instead of* 121 *?*

4. *A population has mean* 128 *and standard deviation* 22 .

   (a) *Find the mean and standard deviation of $\bar{X}$ for samples of size* 36 .

   (b) *Find the probability that the mean of a sample of size* 36 *will be within* 10 *units of the population mean, that is, between* 118 *and* 138 .

5. *A population has mean* $1,542$ *and standard deviation* 246 *.*

   (a) *Find the mean and standard deviation of* $\bar{X}$ *for samples of size* 100 *.*

   (b) *Find the probability that the mean of a sample of size* 100 *will be within* 100 *units of the population mean, that is, between* $1,442$ *and* $1,642$ *.*

Exercice 2.2 (Basics on confidence region)**.** *We ask you to build confidence interval using the following information*

1. *A random sample is drawn from a population of known standard deviation* 11.3*. Construct a* 90% *confidence interval for the population mean based on the information given (not all of the information given need be used).*

   (a) $n = 36, \bar{x} = 105.2$ *and* $s = 11.2$

   (b) $n = 100, \bar{x} = 105.2$ *and* $s = 11.2$

2. *A random sample is drawn from a population of an unknown standard deviation. Construct a* 98% *confidence interval for the population mean based on the information given (not all of the information given need be used).*

   (a) $n = 225, \bar{x} = 92$ *and* $s = 8.4$

   (b) $n = 64, \bar{x} = 92$ *and* $s = 8.4$

3. *A random sample of size* 256 *is drawn from a population whose distribution, mean, and standard deviation are all unknown. The summary statistics are* $\bar{x} = 1011$ *and* $s = 34$ *.*

   (a) *Construct a* 90% *confidence interval for the population mean* μ

   (b) *Construct a* 99% *confidence interval for the population mean* μ

   (c) *Comment on why one interval is longer than the other.*

Exercice 2.3 (A first application)**.**
The number of trips to a grocery store per week was recorded for a randomly selected collection of households, with the results shown in the table below:

2	2	2	1	4	2	3	2	5	4
2	3	5	0	3	2	3	1	4	3
3	2	1	6	2	3	3	2	4	4

Construct a 95% confidence interval for the average number of trips to a grocery store per week of all households.

Exercice 2.4 (A second application)**.**

The lifetime of a light bulb, given in hours, is represented by a random variable X whose distribution is supposed to be Normal with a standard deviation $Sigma = 400$, the parameter of the mean Mu is unknown.

The measurements of the life span of a batch of 9 bulbs gave the following results:

$$2,000; 1,890; 3,180; 1,990; 2,563; 2,876; 3,098; 2,413; 2,596; 1,876$$

1. *Determine a confidence interval for the average life of a light bulb at the* 90% *level.*
2. *Can we affirm, with a risk of error of* 10%*, that the average life of a light bulb is equal to* 2,500 *hours?*